

REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the American Mathematical Society classification scheme. The 1980 Mathematics Subject Classification (1985 Revision) can be found in the December index volumes of Mathematical Reviews.

26[65L05].—J. C. BUTCHER, *The Numerical Analysis of Ordinary Differential Equations; Runge-Kutta and General Linear Methods*, Wiley, Chichester, 1987, xv + 512 pp., 23 $\frac{1}{2}$ cm. Price \$74.95.

This book is destined to become an indispensable reference for any researcher working on the development or analysis of Runge-Kutta (RK) or General Linear (GL) methods for the numerical solution of Initial-Value Problems (IVP's) for Ordinary Differential Equations (ODE's), and it will be used by the research community as a starting point for further work in this area.

As its title and length suggest, it contains a comprehensive in-depth analysis of RK and GL methods, as well as an excellent summary of results from the theory of ODE's, graphs, and combinatorics needed for their development and analysis. Butcher also includes an extensive bibliography of just under one hundred pages. In addition, the book contains a brief discussion of linear multistep, extrapolation and Taylor series methods. These, though, are not analyzed in depth, but rather are included for completeness in an overview of numerical methods for the solution of IVP's.

Many readers, such as myself, who have not closely followed the rapid development of RK and GL methods in research journals during the past decade will welcome this coherent presentation of the fruits of this labor. Further, it is most appropriate that Butcher himself should have written this book since he has contributed more than any other researcher over the past three decades to the development of RK and GL methods.

The reader, though, may be surprised to learn that a well-written book of this length does not contain everything of importance on such a seemingly specialized topic—but this is indeed the case. In the preface, Butcher alludes to a sequel on “practical issues concerning the design of efficient differential equation software”. This underscores the observation that the current volume is devoted almost exclusively to theoretical issues. These, though, are covered thoroughly in Butcher's elegant, but terse, mathematical style.

Another indication of the extent of this seemingly narrow topic and its impressive rate of development over the past decade is the recent publication of two other books in this field [1], [2]. Although it addresses other closely related topics, the first monograph focuses on the highly specialized subtopic of the nonlinear stability of Runge-Kutta methods, a research area that has blossomed during the past decade.

Butcher covers much of this material in his book as well, but not in as much depth and often from a different perspective. The authors of the second book, on the other hand, survey numerical methods for solving IVP's for ODE's. Although they place greater emphasis on linear multistep and extrapolation methods than Butcher does, the strength of their book, like his, lies in the analysis of RK methods. But unlike Butcher, they consider practical aspects associated with the numerical solution of IVP's, and even include codes and numerical test results in their book. However, as the title suggests, their first volume deals with nonstiff IVP's only: a second volume on stiff problems is currently being written.

Any researcher working on RK or GL methods or any library that aims at having a comprehensive collection of works on the numerical solution of IVP's should acquire all three of these excellent books. Although they overlap to some degree, there are many aspects discussed in one but not the others. Moreover, as already mentioned above, topics treated in two or more of these books are often presented from different perspectives, giving the reader a deeper understanding of the material.

Hairer, Nørsett and Wanner's book is the most suitable of the three for either a text for an advanced course on the numerical solution of IVP's or a practical guide to solving IVP's for scientists and engineers. Its obvious drawback is that it does not treat stiff problems, but this deficiency will be rectified, we hope, with the appearance of their second volume. Dekker and Verwer's book is too specialized for either of these two potential audiences. Butcher's, on the other hand, although not more specialized than Hairer, Nørsett and Wanner's, does not treat practical issues to nearly as great an extent and is aimed more at the RK/GL research fraternity who will study it extensively from cover to cover, rather than at the broader scientific computing community looking for insight into solving practical problems. Hairer, Nørsett and Wanner's book, on the other hand, is directed more towards this broader audience, although RK/GL specialists will find it a very useful reference. It is more accessible to nonexperts in the field than Butcher's, because of the less mathematically sophisticated style in which it is written. This also contributes to the ease one frequently finds in reading a topic of interest in it without having studied the preceding sections, a characteristic not shared by Butcher's book.

The comparison in the preceding paragraph should not be taken as negative criticism of any one of these three books. Each is excellent in its own way, but written with different audiences in mind.

KENNETH R. JACKSON

Computer Science Department
University of Toronto
Toronto, Canada M5S 1A4

1. K. DEKKER & J. G. VERWER, *Stability of Runge-Kutta Methods for Stiff Nonlinear Differential Equations*, CWI Monograph, vol. 2, North-Holland, Amsterdam, 1984.
2. E. HAIRER, S. P. NØRSETT & G. WANNER, *Solving Ordinary Differential Equations I, Nonstiff Problems*, Springer, Berlin, 1987.